

FRACTURING OF HETEROGENEOUS MATERIALS BY THERMALLY  
INDUCED RESIDUAL STRESS WITH SPECIAL REFERENCE TO ROCKS

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Residual internal stress, more or less, exists in most of polycrystalline materials. Temperature change induces internal stress field around the boundary of mechanical inhomogeneity. Inhomogeneous chemical reaction, phase change or plastic deformation causes residual internal stress. High residual elastic stress has been measured in many kinds of rocks by the X-ray diffraction technique (1). The residual elastic strain controls the orientation of fractures in sandstone (2). The quenched metal or glass contains thermally-induced residual stress. The cold working produces stress-induced residual stress.

Many studies have been devoted to the estimation of thermal stress which is induced by temperature gradient in partially heated material. However, internal stress is induced by mechanical inhomogeneity under homogeneous temperature or under small temperature gradient. The effect of residual elastic stress on fracture formation is not known quantitatively. A fracture theory which includes the effect of residual stress is discussed in this study.

Residual Stress around a Spheroidal Inclusion

The author has proposed the Griffith inclusion model of fracture which is a natural extension of the Griffith crack model (3),(4). In the Griffith inclusion model, penny-shaped inclusions are postulated as fracture nuclei instead of Griffith cracks. The Griffith inclusion model, which is important especially under pressure, includes the Griffith crack model as a case of empty inclusion. When material is subjected to temperature change, Griffith inclusions become loci of internal stress.

Edwards (5) has obtained the exact analytical solution for stress distribution in and around a prolate spheroidal-shaped inclusion within an infinite homogeneous elastic surrounding body by the Boussinesq's three function method. In the fracture mechanics, oblate spheroidal-shaped inclusions are more important, because high stress concentration is expected around a so-called penny-shaped inclusion.

The exact solution for distribution of elastic stress in and around an oblate spheroidal-shaped inclusion is obtained from Edwards' solution after a transformation of variable (3). In this solution, it is assumed that a single isolated spheroidal-shaped inclusion exists in an infinite homogeneous elastic body whose elastic properties are different from them of the inclusion, that two media are completely bonded at the boundary, and that stress state is uniform at infinity. The solution for stress distribution is too complicated to be described completely. The highest stress concentration which controls formation of fractures occurs at the tip of penny-shaped inclusion. The following stress ( $\sigma_r$ ) in the radial direction is possibly induced by temperature and stress change at the spheroidal inclusion tip (tensile stress is positive).

$$\sigma_r = - \frac{(1-\nu)G'}{GD} \left\{ 3K'(\epsilon' \Delta T' - \epsilon \Delta T) + \left(1 - \frac{K'}{K}\right) \Delta \sigma \right\} \dots (1)$$

$$D = 2\pi/S + 4(1-\nu)K'/G + 16(1-\nu)/3G \dots (2)$$

where  $S$  is the aspect ratio of the penny-shaped inclusion,  $G$ ,  $K$  and  $\nu$  are rigidity, bulk modulus and Poisson's ratio of the host material, respectively,  $G'$  and  $K'$  are rigidity and bulk modulus of inclusion,  $\epsilon'$  and  $\epsilon$  are the linear thermal expansion coefficients of inclusion and host, respectively,  $\Delta T'$  and  $\Delta T$  are the temperature change of inclusion and host, respectively, and  $\Delta \sigma$  is the change of applied hydrostatic stress. If homogenized temperature ( $T_0$ ) and stress ( $\sigma_0$ ) are defined as a condition that residual internal stress disturbance vanishes:

$$\Delta T' = T' - T_0, \quad \Delta T = T - T_0, \quad \Delta \sigma = \sigma - \sigma_0 \dots (3)$$

where  $T'$  and  $T$  are present temperature of inclusion, present temperature of host and present normal stress, respectively. In many cases, the homogenized temperature and stress are practically temperature and stress immediately before quenching.

The stress ( $\sigma_t$ ) which acts in the host at the inclusion tip in the transverse direction to the equatorial plane of the penny-shaped inclusion is estimated as follows:

$$\sigma_t = \frac{8}{D} \left(1 - \nu \frac{G'}{G}\right) \left\{ 3K'(\epsilon' \Delta T' - \epsilon \Delta T) + \left(1 - \frac{K'}{K}\right) \Delta \sigma \right\} \dots (4)$$

The boundary parting by shrinkage of inclusion or splitting by expansion of inclusion occurs at the inclusion tip.

### Residual Stress Effect on Griffith Inclusion

In the Griffith inclusion model, fracture sources are soft penny-shaped inclusion ( $G'/G$  is small)(4). The possible maximum stress at the boundary of Griffith inclusion is obtained under the existence of residual stress as follows:

$$\sigma_{max} = \frac{1}{2} (\sigma_t + \sqrt{\sigma_t^2 + b^2 \tau^2}) \quad \dots (5)$$

$$b = 8(1 - \frac{G'}{G}) / \left\{ \frac{(2-\nu)\pi}{s} + 4(1-\nu) \frac{G'}{G} \right\} \quad \dots (6)$$

Where  $\tau$  is macroscopic shearing stress along the equatorial plane of inclusion at infinity.

If the fracture initiates where the maximum stress around the Griffith inclusion reaches the ideal tensile strength ( $T_m$ ) of the host material, the fracture criterion under the coexistence of residual stress is obtained as follows:

$$\tau = \frac{2T_m}{b} \sqrt{1 - \sigma_t/T_m} \quad \dots (7)$$

The equation (7) can be modified by using experimentally obtained constants  $\tau_0$  and  $C$ , ( $T \neq T$ ) as follows:

$$\tau = \sqrt{\tau_0 [ \tau_0 - C \{ (1 - K'/K) \sigma + 3K'(\epsilon' - \epsilon) T \} ]} \quad \dots (8)$$

Where  $\tau_0$  is the critical shearing stress at  $T = 0$  and  $\sigma = 0$ .

### Fracturing by Thermal Stress

Although estimation of effective microscopic mechanical properties are very difficult, the effects of temperature change on fracturing can be calculated in cases of some kinds of inclusions. When temperature rises from 20°C to 100°C, theoretically estimated thermal stress in steel is about 7500kg/cm<sup>2</sup> at the maximum at the tip of penny-shaped lead inclusion and about 1800kg/cm<sup>2</sup> at the tip of copper inclusion. If penny-shaped lead or copper inclusions exist in steel, microfractures can be formed by relatively small temperature change. However, stress concentration is low around round-shaped inclusion.

High residual stress exists in glass because of quenching. Nickel sulfide inclusion causes spallation of glass (T. Fujiwara, personal communication). Millerite (NiS) inverts

on heating to the high-temperature form at 396°C, whereas on cooling the inversion occurs 100°C or more below this temperature (5). Millerite needle-shaped crystal expands about 18% in the direction of c-axis at the time of inversion. The retard phase transition of millerite can cause spallation of glass. The large difference of thermal expansion coefficients between quartz and feldspars weakens granitic rock which suffered temperature change. Partial volume expansion by phase transformation of quartz causes drastic weakening of granitic rock at about 573°C. Temperature change which crosses the phase transformation point of quartz weakens granitic rocks and accelerates drilling of hot granitic rocks in geothermal area. Partial volume expansion or reduction of rigidity by melting or other phase change triggers faulting and earthquakes under high pressure at deep parts of earth's crust and mantle. Quantitative estimation of effect of internal stress concentration is essential to rock mechanics, tectonophysics and fracture mechanics of metals, ceramics and composites.

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