

# Statistical evaluation of stability of faults on the basis of Griffith inclusion model

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## 1 INTRODUCTION

Virtually every rock mass contains various kinds of weak zones such as faults, joints, cleavages, layers of soft sediments, altered pockets, weak grains or grain boundaries. The deformation and failure of weak zones cause stress concentration in the intact rocks and often result in the failure of whole rock mass. Thus, the mechanical behavior of weak zones or planes of weakness controls total failure process of rock mass.

The Griffith theory for fracture (Griffith, 1924) is widely accepted as the most reasonable model describing fracture propagation in rocks under tensile or low compressive stress conditions. The pre-existing cracks which are considered in the Griffith theory are dry clean elliptical pores which do not contain any foreign materials. As the parting along planes of weakness easily occurs under tensile stress, clean Griffith cracks can provide virtually adequate model for fracture initiation from planes of weakness in rocks under tensile or atmospheric stress conditions. Under overall compressive stresses, however, the presence of lubricants or obstacles within the pre-existing cracks has significant effect on fracture propagation.

A relatively small plane of weakness in large rock mass can be mechanically analyzed as a penny-shaped inclusion. Penny-shaped inclusions ("Griffith inclusions") have some analogy in mechanical behavior with penny-shaped "clean" cracks (Griffith cracks in the three dimensions). However, the Griffith inclusions are more effective sources for fractures under high compressive stress than Griffith cracks in a narrow sense (Koide, 1972).

## 2 GRIFFITH INCLUSION MODEL

The maximum stress concentration is induced at about the tip of the Griffith inclusion. Koide(1968) has obtained the exact stress distribution around a penny-shaped inclusion within an infinite homogeneous elastic host material by the Boussinesq's three function method. Although this assumption for analysis is very simple, this solution provides an adequate estimation for stress concentration around foreign inclusions within relatively large host material. The stress concentration factor of mode I for a penny-shaped inclusion is obtained as:

$$a = 8(1 - \nu \frac{G'}{G}) / \{ 2\pi s + 4(1 - \nu) \frac{K'}{G} + \frac{16}{3}(1 - \nu) \frac{G'}{G} \} \quad (1)$$

where "s" is the aspect ratio of the penny-shaped inclusion. The aspect ratio is very small for a sharp thin inclusion. "G" and "ν" are rigidity and Poisson's ratio of host rocks, respectively. "G'" and "K'" are rigidity and bulk modulus of the inclusion, respectively. When only normal stress (compressive stress is positive) infinitely far in the host rocks acts perpendicularly to the equatorial plane of penny-shaped inclusion, then, the stress concentration of mode I only occurs at the inclusion tip even under the existence of internal pressure "p" and dilatancy of inclusion "v'" as follows:

$$T_1 = a \{ (1 - K'/K) (\sigma - p) - K'v' \} \quad (2)$$

where "K" is bulk modulus of host rock. The stress concentration factor of mode II is:

$$b = 4(1 - G'/G) / \{ (2 - \nu)\pi s + 4(1 - \nu)G'/G \} \quad (3)$$

Where only shearing stress infinitely far in the host rock acts parallel to the

equatorial plane of the penny-shaped inclusion, the stress concentration of mode II only occurs at about inclusion tip as follows:

$$T_2 = b\tau \quad (4)$$

or

$$T_2 = \frac{4}{(2-\nu)\pi b} (\tau - S') + bS' \quad (\tau > S') \quad (5)$$

where "S'" is the yield stress of the inclusion.

Under the general stress conditions, the maximum local tensile stress (negative in this paper) at about the tip of penny-shaped Griffith inclusion can be obtained as the combination of the stress concentrations of mode I and mode II as follows:

$$T = (1/2) (T_1 - \sqrt{T_1^2 + 4T_2^2}) \quad (6)$$

Other stress components affect very little on the stress concentration around the Griffith inclusion except for the case of extreme stress conditions.

In the Griffith theory (Griffith, 1924), it is assumed that crack propagation occurs when the maximum local stress reaches a value equal to the inherent tensile strength " $T_m$ " of the host material. This Griffith criterion is very simple and practically adequate for rocks in which small tensile cracks originate at first under deformation (Koide et al., 1970). From the equation (6), the critical tangential stress for fracture initiation from a penny-shaped Griffith inclusion is obtained as follows:

$$T_2 = T_m \sqrt{1 + T_1/T_m} \quad (7)$$

If the yield stress is enough high, fractures initiate in the host rock at about the tip of inclusion. When the bulk modulus of inclusion is very low and the effect of dilatancy of inclusion is negligible, the equation (7) can be simplified as follows:

$$\tau = (T_m/b) \sqrt{1 + (a/T_m)(\sigma - p)} \quad (8)$$

If  $K'=G'=0$ , then, the equations (8), (1) and (3) give the three dimensional Griffith criterion for a propagation of penny-shaped crack. These equations give a generalized Griffith criteria for fracture propagation from various kinds of Griffith cracks and inclusions.

The rigidity of inclusion "G'" is zero in the above equations for fluid-filled cracks or fluid inclusions. For a penny-shaped fluid inclusion, the stress

concentration factor "b" of mode II is very high but the stress concentration factor "a" of mode I is relatively low due to the incompressibility of the undrained fluid inclusion. Therefore, the generalized Griffith criterion for undrained fluid inclusion is expected to be a very flat parabola on the Mohr's diagram. On the other hand, the generalized Griffith criterion for drained fluid inclusion is similar to the Griffith crack case.

We have only few knowledge on the constitutive properties of the materials which compose the weak zones in rocks. Weak platy minerals such as biotite in granite or soft inclusion rocks such as serpentinite embedded in hard rocks can be source of fracture as Griffith inclusions.

### 3 FAULT GOUGE ZONE AS A GRIFFITH INCLUSION

Active faults are one of the major geologic hazards. The mechanical response of faults to tectonic or artificial stress is most concerned in the designs of large scale construction and underground facilities and in the earthquake prediction. The dilatancy model for earthquake prediction is based on the formation of weakened zone by the concentration of microfractures. Stuart (1974), Brady (1974), Rudnicki and Rice (1975) considered this weakened zone as a weak inclusion and proposed the inclusion model for earthquake prediction. However, their theories are not based on the complete solution of stress distribution around a sharp inclusion under various stress conditions. The field of high stress concentration around tips of sharp penny-shaped inclusions can trigger earthquakes (Koide, 1969; Koide et al., 1970; Koide et al., 1974).

Mogi (1977) has pointed out an important pattern of seismic activity that many small earthquakes occur at early period in the source zone of a would-be large earthquake but activity of small earthquakes diffuse into surrounding area leaving a seismically quiescent central zone. A large earthquake initiates at about an end of this central seismically quiescent oval zone. Mogi (1977) called this pattern of foreshock activity as "a doughnut-shaped pattern".

The doughnut-shaped occurrence of foreshocks suggests that the early microfracturing weakens the fault zone forming seismically incompetent region and that the subsequent plastic deformation of fault gouge increases stress concentration around tips of fault zone to trigger a

large earthquake. The apparent yield stress of active fault zone looks very low. Then, the equation (5) is appropriate rather than the equation (4). The Griffith inclusion criteria for instability of active fault can be obtained from the equation (7) as follows:

$$\tau = S' + (2-\nu)\pi(s/4) (-bS' + T_m \sqrt{1 + T_1/T_m}) \quad (9)$$

The constitutive mechanical properties of fault gouge varies widely with the composition of gouge, stress states, water contents or other conditions (Summers and Byerlee, 1970). The yield stress of fault gouge empirically follows the Coulomb criterion (Jaeger and Cook, 1969). Then the equation (9) can be modified into a practical form as follows:

$$\tau = C_0 + \mu(\sigma - p) + C_1 \sqrt{1 + C_2(\sigma - p - K'v')} \quad (10)$$

where " $C_0$ " is the cohesion and " $\mu$ " is the coefficient of friction. The other coefficients are:

$$C_1 = (2-\nu)\pi s T_m / 4 \quad (11)$$

$$C_2 = a/T_m = 2G/(1-\nu)/K'/T_m \quad (12)$$

The criterion (10) has similar form to the Modified Griffith theory (McClintock and Walsh, 1962). The Modified Griffith theory considers the friction between closed surfaces of a dry Griffith crack, although this Griffith inclusion model is related to the lubricated sliding along fault gouge.

As the dilatancy of dry fault zone increases the tensile stress concentration around tips of fault, the acceleration of dilatancy by shearing of dry fault zone can cause instability of fault. However, the effect of decrease in interstitial fluid pressure in a dilatant wet fault zone could cause the dilatancy hardening (Frank, 1965). The subsequent recover of interstitial fluid pressure by diffusion may trigger a large earthquake.

#### 4 STATISTICAL GRIFFITH INCLUSION MODEL

The above discussion of fracture criteria is mainly concerned to the instability of individual fault or weak layer in rocks. Griffith (1924) originally assumed that the critical Griffith crack is available anywhere and in any direction and that the propagation of a single Griffith crack immediately lead to final failure of the host material. Then, the above Griffith criteria for a crack or inclusion form envelopes beyond which the Mohr's stress

circle cannot grow larger on the Mohr's diagram on " $\sigma$ - $\tau$ " co-ordinates. As the outermost stress circle is constructed on the maximum and minimum stresses, the intermediate stress cannot make any influence on the failure condition. However, Mogi (1971) has shown clear experimental evidences of the influence of intermediate stress on the failure of rocks. The Griffith's original assumption is inadequate for rocks under compressive stress conditions.

The rock mass commonly contains various kinds of weak layers of various shapes and orientations. Nishimatsu (1967) has developed a statistical Griffith theory assuming a statistical distribution on the shape of Griffith cracks. The statistical distribution of shape and direction of Griffith inclusions is analyzed in this study under the assumption that the Griffith inclusions are distributed spatially at random and interactions among inclusions are negligible. A probability density function  $f(s, \alpha, \beta)$  is assumed to represent the statistical distribution of Griffith inclusions in the shape ratio " $s$ " and in the orientations " $\alpha$ " and " $\beta$ " on the polar co-ordinates. Then, the probability that unstable Griffith inclusions are contained in a rock mass of volume " $v$ " is:

$$F(\sigma^*) = v \int_0^{2\pi} \int_0^\pi \int_0^s f(s, \alpha, \beta) \sin(\alpha) ds d\alpha d\beta \quad (13)$$

where " $s$ " is the shape ratio of the critical inclusion that can be calculated from the Griffith inclusion criteria as a function of stresses. " $\sigma^*$ " is the general state of stress. In the general case, the equation is complicated. However, the apparent rigidity and/or yield stress is extremely low when the fault gouge contains the expanding clays such as montmorillonite (Summers and Byerlee, 1970). Water and other fluid, which is confined within the fault zone during deformation, drastically reduce the strength of fault gouge. This situation is highly possible in the underground rocks. Then, the effect of normal stress is very small in the Griffith inclusion criteria. The critical shape ratio of inclusion can be obtained from the following simple equation as a function of shearing stress only.

$$s' = \frac{4\tau}{(2-\nu)\pi T_m} \quad (14)$$

For the sake of simplicity, directional distribution of inclusions is assumed to be uniform. Then, the probability density function can be represented by the shape

ratio only. As only very sharp inclusions could be candidates for fracture loci, the shape ratio is extremely small. The probability density function can be approximated to the power function of the shape ratio. As the simplest case, the probability density function is assumed to be proportional to the shape ratio when "s" is very small. Then, the simplest representation of probability of number of unstable Griffith inclusion is obtained as follows:

$$\begin{aligned}
 F(\delta) &= \iiint s^1 A' s \alpha_1 ds d\alpha d\beta = AV \iiint \tau^2 \alpha_1 d\alpha d\beta \\
 &= AV \iiint \{ \sigma_1^2 \alpha_2^2 + \sigma_2^2 \alpha_1^2 \beta_2^2 + \sigma_3^2 \alpha_1^2 \beta_1^2 \\
 &\quad - (\sigma_1 \alpha_2^2 + \sigma_2 \alpha_1^2 \beta_2^2 + \sigma_3 \alpha_1^2 \beta_1^2)^2 \} \alpha_1 d\alpha d\beta \\
 &= (2/15) \pi AV \{ (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \}
 \end{aligned}
 \tag{15}$$

where  $\sigma_1$   $\sigma_2$   $\sigma_3$  are principal stresses, and  $\alpha_1 = \sin\alpha$ ,  $\alpha_2 = \cos\alpha$ ,  $\beta_1 = \sin\beta$ ,  $\beta_2 = \cos\beta$ . As the number of unstable Griffith inclusions is highly related to the condition for failure of rock mass (Nishimatsu, 1967), the failure criterion for faulted rock mass would be a function of the octahedral shear stress. Mogi (1971) has proposed an extended von Mises criterion from the results of general triaxial compression test. The Mogi's extended von Mises criterion is the function of the octahedral shear stress " $\tau_{oct}$ " with additional effect of normal stresses as follows:

$$\tau_{oct} = f'(\sigma_1 + \sigma_3)
 \tag{16}$$

The density of microfractures and their propagating distances directly affect the failure of rock mass. As fracture propagation distances would be affected by the normal stresses, the statistical model of Griffith inclusion suggests that the extended von Mises criterion is applicable to the faulted rocks or many other rocks which contain various weak inclusions.

## 5 CONCLUSION

The Griffith inclusion model for fracture initiation has been discussed on the basis of solution of stress field around tips of penny-shaped inclusion. The Griffith inclusion criteria are natural extension of the Griffith model. The fault propagation can be described adequately by the Griffith inclusion model. The dilatancy instability of earthquake source zone and doughnut-like pattern of foreshock activity reflect failure process of active fault gouge zone. The statistical model of

Griffith inclusions suggests that the extended von Mises criterion is applicable to rocks which commonly contain various weak zones.

## 6 REFERENCES

- Brady, B.T. 1974, Theory of earthquakes. I., Pure & Appl. Geophys. 112:701-725.
- Frank, F.C. 1965, On dilatancy in relation to seismic sources, Rev. Geophys. 3: 485-503.
- Griffith, A.A. 1924, The theory of rupture, Proc. 1st Int. Congr. Appl. Mech., Delft, 54-63.
- Jaeger, J.C. & Cook, N.G.W. 1969, Fundamentals of Rock Mechanics, London, 513p.
- Koide, H. 1968, A tectonophysical study of the initiation and development of fracture in rocks, Dr. Thesis, Univ. Tokyo.
- Koide, H. 1969, Initiation and development of fractures in rocks, Gansekibussai Kenkyukai, 7: 1-16, (in Japanese).
- Koide, H. 1972, Fracture initiation in brittle polycrystalline material such as rocks, Mechanical Behavior of Materials, 4: 455-463.
- Koide, H., Hoshino, K. & Nagumo, S. 1970, Microscopic mechanism of the development of rock failure, Rock Mech. in Japan, 1: 62-64.
- Koide, H., Hoshino, K. & Inami, K. 1976, Fault models based on an extended Griffith's theory, Bull. Geol. Surv. Japan, 25: 89-73 (in Japanese).
- McClintok, F.A. & Walsh, J.B. 1962, Friction on Griffith cracks in rocks under pressure, Proc. 4th U.S. Nat. Congr. Appl. Mech., 1015-1021.
- Mogi, K. 1971, Fracture and flow of rocks under high triaxial compression, Jour. Geophys. Res., 76:1255-1269.
- Mogi, K., 1977, Dilatancy of rocks under general triaxial stress states with special reference to earthquake precursors, J. Phys. Earth., 25: Suppl., S203-S217.
- Rudnicki, J.R. & Rice, J.R. 1975, Conditions for the localization of deformation in pressure sensitive dilatant materials, J. Mech. Phys. Solids, 23: 371-394.
- Stuart, W.D. 1974, Diffusionless dilatancy model for earthquake precursors, Geophys. Res. Lett. 1: 261-264.
- Summers, R. & Byerlee, J. 1977, A note on the effect of fault gouge composition on the stability of frictional sliding, Int. J. Rock Mech. Min. Sci., 14:155-160.

PROCEEDINGS OF THE INTERNATIONAL SYMPOSIUM ON WEAK ROCK  
TOKYO / 21-24 SEPTEMBER 1981

# Weak Rock

## *Soft, Fractured and Weathered Rock*

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