

Hydraulic fracturing and stress distribution around a pressure shaft and spheroidal cavern

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ABSTRACT: The hydraulic fracturing of rocks is the major problem for pressure shaft, tunnels and storage caverns. The hydraulic fracturing method for in-situ stress measurements usually supposes the formation of axial fractures around a cylindrical borehole subjected to internal fluid pressures. However, the fractures curve out vertical to the minimum regional stress in short distances from the borehole. Transverse fractures are formed under extreme stress conditions. The exact stress distributions around a cylindrical hole or spheroidal cavity with internal fluid pressures were obtained from an analytical solution based on the three-dimensional elastic theory under various regional stress conditions. The combined effects of internal fluid pressure and regional deviatoric stresses are discussed for adequate control of induced fractures around pressure shafts, tunnels and caverns subjected to internal fluid pressures.

1 INTRODUCTION

The hydraulic fracturing phenomena were first realized from quick loss of water during drilling of oil well (Hubbert, Willis, 1957). The hydraulic fracturing was applied to the in-situ measurement of stress in rock.

The formation of fluid conduit by hydraulic fracturing is vital to the extraction of geothermal energy from hot dry rocks. The pressure tunnel and underground storage cavern for oil could be fractured by the effect of fluid intrusion into the wall rock.

However, the mechanism of hydraulic fracturing has not been understood enough to control the path and direction of fractures. The hydraulic fracturing method for in-situ stress measurement usually assumes the formation of fractures along an axial plane around a cylindrical borehole due to internal fluid pressure. However, hydraulic fractures curve out into the vertical direction to the minimum regional stress in short distances from the wall of borehole. Transverse fractures could be formed under some stress conditions.

In this paper, the orientation of hydraulic fractures is discussed on the basis of an exact analytical solution of

three dimensional elastic theory around a spheroidal cavity and around a cylindrical hole.

2. STRESS AROUND A SPHEROIDAL CAVITY

Using the Boussinesq's three function method of the three dimensional elastic theory, Edwards (1951) obtained an exact solution for the distribution of stress around a prolate spheroidal cavity in an infinite homogeneous, isotropic elastic body which is under an arbitrary uniform state of stress at infinite distances from the cavity.

Although the Edwards' solution allows us the exact estimation of stress distribution around a spheroidal cavity, the solution is too complicated and hard to be calculated.

In this study, the Edwards' solution is modified into a more convenient form for estimation. This equations give the estimation of effect of internal fluid pressure in the cavity, too.

Let the Cartesian co-ordinate system O-xyz be introduced with O at the center of the spheroidal cavity and O-z coincident with the axis of revolution of the cavity (Fig.1). Since the state of stress at infinity is uniform, O-xy can be

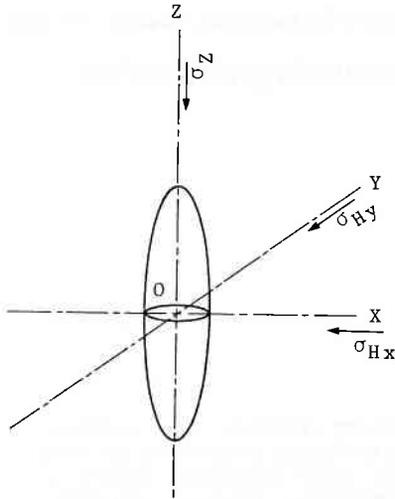


Fig.1 Spheroidal cavity

oriented so that the shearing stress at infinity is zero. The stress distribution throughout the elastic body can be obtained as a linear combination of the solution to the following loading conditions (tension is positive in this paper):

Case 1: (isotropic state of stress)
 $\sigma_x = \sigma_y = \sigma_z = \sigma_i, \tau_{xy} = \tau_{yz} = \tau_{zx} = 0$ (1)

Case 2: (plane isotropic state of stress)
 $\sigma_x = \sigma_y = \sigma_n, \sigma_z = \tau_{xy} = \tau_{yz} = \tau_{zx} = 0$ (2)

Case 3: (plane state of pure shearing stress)
 $\sigma_x = -\sigma_y = \sigma_d, \sigma_z = \tau_{xy} = \tau_{yz} = \tau_{zx} = 0$ (3)

Case 4: (plane state of pure shearing stress)
 $\tau_{zx} = \tau_x, \sigma_x = \sigma_y = \sigma_z = \tau_{xy} = \tau_{yz} = 0$ (4)

Case 5: (plane state of pure shearing stress)
 $\tau_{yz} = \tau_y, \sigma_x = \sigma_y = \sigma_z = \tau_{xy} = \tau_{zx} = 0$ (5)

The effect of internal fluid pressure in the cavity (pressure is negative in this paper) is also estimated:

Case 6: (internal fluid pressure in cavity)
 $P_i = P, \sigma_x = \sigma_y = \sigma_z = \tau_{xy} = \tau_{yz} = \tau_{zx} = 0$ (6)

In the classical three-function approach of the theory of elasticity, the general solution of the displacement equilibrium equations in the absence of body forces is taken as:

$$[u, v, w] = \frac{1}{2G} \text{grad} \phi$$

$$[u, v, w] = \frac{1}{G} \text{curl} [0, 0, \theta] \quad (7)$$

$$[u, v, w] = \frac{z}{2G} \text{grad} \lambda - [0, 0, \frac{3-4\nu}{2G} \lambda]$$

$$\Delta \phi = \Delta \theta = \Delta \lambda = 0$$

where G is the rigidity and ν is the Poisson's ratio of the surrounding elastic body. Any solution of the displacement-equilibrium equations may be represented as a linear combination of these solutions provided the harmonic functions (ϕ, θ, λ) are chosen properly. The prolate spheroidal co-ordinate system is defined by the equation of transformation:

$$\begin{aligned} x &= \sinh \alpha \sin \beta \cos \gamma = \bar{q} p \cos \gamma \\ y &= \sinh \alpha \sin \beta \sin \gamma = \bar{q} p \sin \gamma \\ z &= \cosh \alpha \cos \beta = \bar{q} p \end{aligned} \quad (8)$$

where:

$$\begin{aligned} q &= \sinh \alpha, \bar{q} = \cosh \alpha = \sqrt{1+q^2} \\ p &= \cos \beta, \bar{p} = \sin \beta = \sqrt{1-p^2} \end{aligned} \quad (9)$$

As $q=q_0$ and $\alpha=q_0$ at the wall of the prolate cavity, the longest diameter of the cavity is equal to $2q_0$ along the z -axis and the shortest diameter is equal to $2q_0$ in the equator of cavity.

Suitable harmonic functions should be chosen on the spheroidal co-ordinate and coefficients should be determined to fulfill the boundary conditions that the stress is uniform at infinity and the normal stress is constant on the cavity wall ($q=q_0$). As the process of calculation is the essentially same as that of Edwards (1951), the newly modified final equations, only, are described in this paper.

For convenience, the following auxiliary functions are introduced:

$$\begin{aligned} h^2 &= \frac{1}{q^2 + p^2} \\ h_0^2 &= \frac{1}{q_0^2 + p^2} \\ Q &= -\bar{q}^2 + q^2 \bar{q} \coth^{-1} \bar{q} = -\bar{q}^2 - \frac{q}{2} \log \frac{\bar{q}-1}{\bar{q}+1} \\ Q_0 &= -q_0^2 + q_0^2 \bar{q} \coth^{-1} \bar{q}_0 = -q_0^2 - \frac{q_0}{2} \log \frac{q_0-1}{q_0+1} \end{aligned} \quad (10)$$

2.1 Case 1: isotropic state of stress

the surrounding elastic body is under an isotropic uniform stress (σ_i) at infinity:

as $q \rightarrow \infty$

$$\sigma_x, \sigma_y, \sigma_z \rightarrow \sigma_i$$

$$\tau_{xy}, \tau_{yz}, \tau_{zx} \rightarrow 0$$

(11)

or equivalently, as $q \rightarrow \infty$

$$\sigma_\alpha, \sigma_\beta, \sigma_\gamma \rightarrow \sigma_i$$

$$\tau_{\alpha\beta}, \tau_{\beta\gamma}, \tau_{\gamma\alpha} \rightarrow 0$$

(12)

The boundary stress condition at the cavity wall is : at $q=q_0$

$$\sigma_\alpha = \tau_{\alpha\beta} = \tau_{\alpha\gamma} = 0$$

(13)

At an arbitrary point in the surrounding rock, stresses are obtained on the spheroidal co-ordinate (α, β, γ) as follows:

$$\sigma_\alpha = \sigma_i + \frac{\sigma_i}{D_1} \{ a_{11h} S_{1\alpha} + a_{21h} S_{2\alpha} + (q^2 - q_0^2) a_{11h} S_{3\alpha} \}$$

$$\sigma_\beta = \sigma_i + \frac{\sigma_i}{D_1} \{ a_{11h} S_{1\beta} + a_{21h} S_{2\beta} + (q^2 - q_0^2) a_{11h} S_{3\beta} \}$$

$$\sigma_\gamma = \sigma_i + \frac{\sigma_i}{D_1} \{ a_{11h} S_{1\gamma} + a_{21h} S_{2\gamma} + (q^2 - q_0^2) a_{11h} S_{3\gamma} \}$$

(14)

$$\tau_{\alpha\beta} = \frac{\sigma_i}{D_1} \{ a_{11h} S_{1\alpha\beta} + a_{21h} S_{2\alpha\beta} + (q^2 - q_0^2) a_{11h} S_{3\gamma} \}$$

$$\tau_{\beta\gamma} = \tau_{\alpha\gamma} = 0$$

where the coefficients are as follows:

$$D_1 = 4 \{ -(1+\nu)Q_0^2 - Q_0 - q_0^2 (2+3Q_0) \}$$

$$a_{11h} = 2(2+3Q_0) \bar{q}_0 \bar{q}_0^2$$

(15)

$$a_{21h} = -2 \{ 1 + (1+\nu)Q_0 \} \bar{q}_0 \bar{q}_0^2$$

The basic solutions are as follows:

$$S_{1\alpha} = \frac{q}{q^2} \{ h^2 p^2 \{ 1 + (1+\nu)Q \} + 2 \}$$

$$S_{1\beta} = \frac{h^2}{q} \{ -\bar{q}^2 + p^2 (1+\nu)Q \}$$

(16)

$$S_{1\gamma} = -\frac{q}{q^2} \{ 1 - \frac{1+\nu}{\bar{q}^2} Q - 2\nu h^2 p^2 \}$$

$$S_{1\alpha\beta} = \frac{h^2 p p}{q} \{ 1 + (1+\nu)Q \}$$

$$S_{2\alpha} = \frac{1}{q q^2} \{ -2Q + h^2 \bar{q}^2 p^2 (2+3Q) \}$$

$$S_{2\beta} = \frac{1}{q q^2} \{ Q - h^2 \bar{q}^2 p^2 (2+3Q) \}$$

(17)

$$S_{2\gamma} = \frac{Q}{q q^2}$$

$$S_{2\alpha\beta} = h^2 p \frac{\bar{p}}{q} (2+3Q)$$

$$S_{3\alpha} = -\frac{h^2 \bar{q}}{q^2} h^4 \bar{q}$$

$$S_{3\beta} = h^4 \bar{q}$$

(18)

$$S_{3\gamma} = \frac{h^2 \bar{q}}{q^2}$$

$$S_{3\alpha\beta} = -h^4 p \frac{\bar{p}}{q}$$

2.2 Case2: plane isotropic state of stress

The surrounding elastic body is under a plane isotropic stress (σ_m) at infinity:

as $q \rightarrow \infty$,
 $\sigma_x, \sigma_y \rightarrow \sigma_m$

$$\sigma_z, \tau_{xy}, \tau_{yz}, \tau_{zx} \rightarrow 0$$

(19)

or equivalently; as $q \rightarrow \infty$,

$$\sigma_\alpha \rightarrow \sigma_m h^2 q^2 p^2$$

$$\sigma_\beta \rightarrow \sigma_m h^2 q^2 p^2$$

$$\sigma_\gamma \rightarrow \sigma_m$$

(20)

$$\tau_{\alpha\beta} \rightarrow \sigma_m h^2 q p \bar{p}$$

$$\tau_{\beta\gamma}, \tau_{\alpha\gamma} \rightarrow 0$$

The boundary stress condition at the cavity wall is: at $q=q_0$,

$$\sigma_\alpha = \tau_{\alpha\beta} = \tau_{\alpha\gamma} = 0$$

(21)

Then, stresses are obtained at an arbitrary point in the surrounding rock on the spheroidal co-ordinate (α, β, γ) as follows:

$$\sigma_\alpha = \sigma_m h^2 q^2 p^2 + \frac{\sigma_m}{D_1} \{ a_{11} S_{1\alpha} + a_{21} S_{2\alpha} + (q^2 - q_0^2) a_{11} S_{3\alpha} \}$$

$$\sigma_\beta = \sigma_m h^2 q^2 p^2 + \frac{\sigma_m}{D_1} \{ a_{11} S_{1\beta} + a_{21} S_{2\beta} + (q^2 - q_0^2) a_{11} S_{3\beta} \}$$

$$\sigma_\gamma = \sigma_m + \frac{\sigma_m}{D_1} \{ a_{11} S_{1\gamma} + a_{21} S_{2\gamma} + (q^2 - q_0^2) a_{11} S_{3\gamma} \}$$

(22)

$$\tau_{\alpha\beta} = \sigma_m h^2 q p \bar{p} + \frac{\sigma_m}{D_1} \{ a_{11} S_{1\alpha\beta} + a_{21} S_{2\alpha\beta} + (q^2 - q_0^2) a_{11} S_{3\alpha\beta} \}$$

$$\tau_{\beta\gamma} = \tau_{\alpha\gamma} = 0$$

The coefficients are determined as follows:

$$\begin{aligned} a_{11} &= 4q_0 q_0 \bar{q}_0^2 \\ a_{21} &= 4q_0^3 \bar{q}_0^2 \end{aligned} \quad (23)$$

The basic solutions are the same as in the case 1.

Case 2.3: plane state of pure shearing stress.

The uniform deviatoric stresses perpendicular to the axis of revolution (z-axis) of spheroidal cavity are applied to the surrounding rock at infinity: as $q \rightarrow \infty$

$$\sigma_z, \tau_{xy}, \tau_{yz}, \tau_{zx} \rightarrow 0 \quad (24)$$

or equivalently, as $q \rightarrow \infty$

$$\begin{aligned} \sigma_\alpha &\rightarrow \sigma_d h^2 \frac{q^2}{q^2 p^2} \cos 2\gamma \\ \sigma_\beta &\rightarrow \sigma_d h^2 q^2 p^2 \cos 2\gamma \\ \sigma_\gamma &\rightarrow -\sigma_d \cos 2\gamma \end{aligned} \quad (25)$$

$$\tau_{\alpha\beta} \rightarrow \sigma_d h^2 q \bar{q} p p \cos 2\gamma$$

$$\tau_{\alpha\gamma} \rightarrow -\sigma_d h q \bar{q} p \sin 2\gamma$$

$$\tau_{\beta\gamma} \rightarrow -\sigma_d h q p \sin 2\gamma$$

The boundary stresses at the cavity wall is: at $q = q_0$,

$$\sigma_\alpha = \tau_{\alpha\beta} = \tau_{\alpha\gamma} = 0 \quad (26)$$

Stresses are obtained at an arbitrary point in the surrounding rock on the spheroidal co-ordinate (α, β, γ) as follows:

$$\sigma_\alpha = \sigma_d h^2 \frac{q^2}{q^2 p^2} \cos 2\gamma - \frac{\sigma_d h^2 q \bar{q} p^2 \cos 2\gamma}{q^4 D_2} [q^2 V + 3(q^2 - q_0^2) \{12 + 8h^2 q^2 + 2q^2(2+3Q)\}]$$

$$\begin{aligned} \sigma_\beta &= \sigma_d h^2 q^2 p^2 \cos 2\gamma + \frac{\sigma_d \cos 2\gamma}{q q^4 D_2} \{h^2 q^2 p^2 \{-q^2 V \\ &+ 48q^2 - 3(q^2 - q_0^2) [4 + 8h^2 q^2 + 2q^2(2+3Q)]\} \\ &- 48\nu q^2 + 24(q^2 - q_0^2) h^2 q^2\} \end{aligned}$$

$$\begin{aligned} \sigma_\gamma &= -\sigma_d \cos 2\gamma + \frac{\sigma_d \cos 2\gamma}{q q^4 D_2} [q^2 V \\ &+ 48q^2 (\nu h^2 q^2 p^2 - 1) + 3(q^2 - q_0^2) \{4 \\ &+ 8h^2 q^2 p^2 + 2q^2(2+3Q)\}] \end{aligned} \quad (27)$$

is: at $q = q_0$

$$\sigma_\alpha = \tau_{\alpha\beta} = \tau_{\alpha\gamma} = 0 \quad (31)$$

Stresses are obtained at an arbitrary point in the surrounding rock on the spheroidal co-ordinate (α, β, γ) as follows:

$$\begin{aligned} \sigma_\alpha &= 2h^2 q p p (\tau_x \cos \gamma + \tau_y \sin \gamma) \left\{ \bar{q} - \frac{1}{q^4 D_4} \{- (2 - \nu) q^2 q + q^2 \bar{q}^2 (2+3Q) + (q^2 - q_0^2) [1 - q^2(2+3Q) + h^2 q^2]\} \right\} \end{aligned}$$

$$\sigma_\beta = -2h^2 q p p (\tau_x \cos \gamma + \tau_y \sin \gamma) \left\{ \bar{q} + \frac{1}{q^2 D_4} [2 + (2 - \nu) q - \bar{q}^2 (2+3Q) + (q^2 - q_0^2) (2+3Q - h^2)] \right\}$$

$$\tau_{\alpha\beta} = \sigma_d h^2 q \bar{q} p p \cos 2\gamma \left\{ 1 + \frac{1}{q q^4 D_2} \{-q^2 V + 3(q^2 - q_0^2) [4 + 8h^2 q^2 - 2q^2(2+3Q)]\} \right\}$$

$$\tau_{\alpha\gamma} = -\sigma_d h q p \sin 2\gamma \left\{ 1 + \frac{1}{q q^4 D_2} \{-q^2 V + 3(q^2 - q_0^2) [12 - 2q^2(2+3Q)]\} \right\}$$

$$\tau_{\beta\gamma} = -\sigma_d h q p \sin 2\gamma \left\{ 1 + \frac{1}{q q^4 D_2} \{-q^2 V + 48(1-\nu)q^2 - 3(q^2 - q_0^2) [4 + 2q^2(2+3Q)]\} \right\}$$

where,

$$q_0^2 q_0 D_2 = 6 \{8(1-\nu) - q_0^2(2+3Q_0) + (7-8\nu)Q_0\}$$

$$V = 6 \{8(1-\nu) - q^2(2+3Q) + (7-8\nu)Q\} \quad (28)$$

2.4 Cases 4 and 5: pure shearing stress

The cases 4 and 5 are essentially same stress conditions where one of pure shearing stresses is parallel to the axis of revolution (z-axis) of the spheroidal cavity. The combination of cases 4 and 5 is: as $q \rightarrow \infty$,

$$\tau_{zx} \rightarrow \tau_x$$

$$\tau_{zy} \rightarrow \tau_y \quad (29)$$

$$\sigma_x, \sigma_y, \sigma_z, \tau_{xy} \rightarrow 0$$

or equivalently, as $q \rightarrow \infty$,

$$\sigma_\alpha \rightarrow 2h^2 q p p (\tau_x \cos \gamma + \tau_y \sin \gamma)$$

$$\sigma_\beta \rightarrow 2h^2 q \bar{q} p p (\tau_x \cos \gamma + \tau_y \sin \gamma)$$

$$\sigma_\gamma \rightarrow 0 \quad (30)$$

$$\tau_{\alpha\beta} \rightarrow 2h^2 (q^2 p^2 - \bar{q}^2 p^2) (\tau_x \cos \gamma + \tau_y \sin \gamma)$$

$$\sigma_{\alpha\gamma} \rightarrow -h q p (\tau_x \sin \gamma - \tau_y \cos \gamma)$$

$$\sigma_{\beta\gamma} \rightarrow h q p (\tau_x \sin \gamma - \tau_y \cos \gamma)$$

The boundary stresses at the cavity wall

$$\sigma_y = \frac{h^2 p \bar{p} (\tau_x \cos \gamma - \tau_y \sin \gamma)}{q^3 D_4} \{-4\nu q^2 + 2(q^2 - q_0^2)\} \quad (32)$$

$$\tau_{\alpha\beta} = h^2 (q^2 p^2 - \bar{q}^2 \bar{p}^2) (\tau_x \cos \gamma + \tau_y \sin \gamma) \left\{ 1 + \frac{1}{qq^2 D_4} \left\{ (2-\nu)q - \bar{q}^2 (2+3Q) + (q^2 - q_0^2) (2+3Q) + h^2 \right\} \right\} - \frac{(q^2 - q_0^2) (\tau_x \cos \gamma + \tau_y \sin \gamma)}{qq^2 D_4} \{ 2 - (2q^2 + 1)h^2 \}$$

$$\tau_{\alpha\gamma} = -hqp (\tau_x \sin \gamma - \tau_y \cos \gamma) \left\{ 1 - \frac{1}{qq^2 D_4} \left\{ -(2-\nu)q^2 + q^2 \bar{q}^2 (2+3Q) - (q^2 - q_0^2) (2+q^2 (2+3Q)) \right\} \right\}$$

$$\tau_{\beta\gamma} = hqp (\tau_x \sin \gamma - \tau_y \cos \gamma) \left\{ 1 - \frac{1}{qq^2 D_4} \left\{ -(2(1-\nu) - (2-\nu)q + \bar{q}^2 (2+3Q) - (q^2 - q_0^2) (2+3Q)) \right\} \right\}$$

$$\text{where, } q_0^2 q_0 D_4 = -(2-\nu)q_0 + q_0^2 (2+3Q_0) \quad (33)$$

2.5 Case 6: internal fluid pressure

Although the stress distribution around a spheroidal cavity under any kind of macroscopically uniform state of stress can be obtained from a linear combination of solutions of cases 1,2,3,4 and 5, the internal fluid pressure in a spheroidal cavity, also, induces the stress disturbance field around the cavity. Where the internal fluid pressure (P) pushes the inner wall of a spheroidal cavity:

$$\text{at } q=q_0, \sigma_\alpha = P$$

$$\tau_{\alpha\beta} = \tau_{\alpha\gamma} = 0 \quad (34)$$

The stress condition of the surrounding rock infinitely far from the cavity is:

$$\text{as } q \rightarrow \infty, \sigma_x, \sigma_y, \sigma_z, \tau_{xy}, \tau_{yz}, \tau_{zx} \rightarrow 0 \quad (35)$$

or

$$\text{as } q \rightarrow \infty, \sigma_\alpha, \sigma_\beta, \sigma_\gamma, \tau_{\alpha\beta}, \tau_{\beta\gamma}, \tau_{\alpha\gamma} \rightarrow 0 \quad (36)$$

Stresses at an arbitrary point in the surrounding rock are obtained as follows:

$$\sigma_\alpha = \frac{-P}{D_1} \{ a_{1h} S_{1\alpha} + a_{2h} S_{2\alpha} + (q^2 - q_0^2) a_{1h} S_{3\alpha} \}$$

$$\sigma_\beta = \frac{-P}{D_1} \{ a_{1h} S_{1\beta} + a_{2h} S_{2\beta} + (q^2 - q_0^2) a_{1h} S_{3\beta} \}$$

$$\sigma_\gamma = \frac{-P}{D_1} \{ a_{1h} S_{1\gamma} + a_{2h} S_{2\gamma} + (q^2 - q_0^2) a_{1h} S_{3\gamma} \} \quad (37)$$

$$\tau_{\alpha\beta} = \frac{-P}{D_1} \{ a_{1h} S_{1\alpha\beta} + a_{2h} S_{2\alpha\beta} + (q^2 - q_0^2) a_{1h} S_{3\alpha\beta} \}$$

$$\tau_{\alpha\gamma} = \tau_{\beta\gamma} = 0$$

where the coefficients and basic solutions are all the same as in the Case 1.

3. STRESS AROUND A CYLINDRICAL HOLE

A cylindrical hole such as a borehole, shaft or circular tunnel can be considered as an extreme case of prolate spheroidal cavity whose aspect ratio (q_0/q_0) is infinite. The solutions in the chapter 2 are much simplified when q_0 and q approach to zero. The cylindrical co-ordinate can be used instead of the spheroidal co-ordinate by substituting $\rho \equiv q/q_0$ as follows (Fig.2):

$$\begin{aligned} x &= \rho \cos \gamma \\ y &= \rho \sin \gamma \\ z &= z \end{aligned} \quad (38)$$

where $\rho=1$ at the wall of cylindrical hole. Stresses at arbitrary point in the surrounding rock are obtained on the cylindrical co-ordinate by modification of solutions in the chapter 2 as follows:

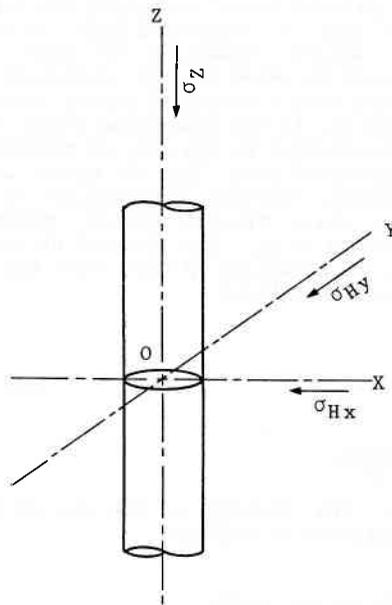


Fig.2 Cylindrical hole

$$\sigma_\rho = (\sigma_i + \sigma_m) \left(1 + \frac{1}{\rho^2}\right) + \frac{P}{\rho^2} + \left(1 - \frac{1}{\rho^2}\right) \left(1 - \frac{3}{\rho^2}\right) \sigma_d \cos 2\gamma$$

$$\sigma_z = \sigma_i - \frac{4\nu}{\rho^2} \sigma_d \cos 2\gamma$$

$$\sigma_\gamma = (\sigma_i + \sigma_m) \left(1 + \frac{1}{\rho^2}\right) - \frac{P}{\rho^2} - \left(1 + \frac{3}{\rho^4}\right) \sigma_d \cos 2\gamma \quad (39)$$

$$\tau_{\rho z} = \left(1 - \frac{1}{\rho^2}\right) (\tau_x \cos \gamma + \tau_y \sin \gamma)$$

$$\tau_{\rho\gamma} = -\left(1 + \frac{3}{\rho^2}\right) \left(1 - \frac{1}{\rho^2}\right) \sigma_d \sin 2\gamma$$

$$\tau_{z\gamma} = \left(1 + \frac{1}{\rho^2}\right) (\tau_y \cos \gamma - \tau_x \sin \gamma)$$

At the wall of cylindrical hole, ($\rho=1$), the stresses are as follows:
 $\sigma_\rho = P$

$$\sigma_z = \sigma_i - 4\nu \sigma_d \cos 2\gamma$$

$$\sigma_\gamma = 2(\sigma_i + \sigma_m) - P - 4\sigma_d \cos 2\gamma \quad (40)$$

$$\tau_{z\gamma} = 2(\tau_y \cos \gamma - \tau_x \sin \gamma)$$

$$\tau_{\rho z} = \tau_{\rho\gamma} = 0$$

The primary normal stress to the direction of axis of cylindrical hole (Z-axis) is σ_N before excavation of hole. The x-axis can be taken to the direction of the plane maximum principal stress (minimum compression) in the transverse plane that is perpendicular to the axis of revolution of cylindrical hole. Then the plane maximum stress (minimum compression) is σ_{Hx} and the plane minimum stress (maximum) compression is σ_{Hy} . The stresses at infinity in the solutions in this paper can be rewritten as follows:
 $\sigma_i = \sigma_N$

$$\sigma_i + \sigma_m = \frac{(\sigma_{Hx} + \sigma_{Hy})}{2} \quad (41)$$

$$\sigma_d = \frac{(\sigma_{Hx} - \sigma_{Hy})}{2}$$

Then, the stresses at the wall of hole are rewritten as follows:
 $\sigma_\rho = P$

$$\sigma_z = \sigma_N - 2\nu (\sigma_{Hx} - \sigma_{Hy}) \cos 2\gamma$$

$$\sigma_\gamma = \sigma_{Hx} + \sigma_{Hy} - P - 2(\sigma_{Hx} - \sigma_{Hy}) \cos 2\gamma \quad (42)$$

$$\tau_{z\gamma} = 2(\tau_y \cos \gamma - \tau_x \sin \gamma)$$

$$\tau_{\rho z} = \tau_{\rho\gamma} = 0$$

4. HYDRAULIC FRACTURE ORIENTATION

The tension fractures are formed under a higher tensile stress than the tensile strength (T_m) of material or under a higher effective stress $\sigma - P_p > T_m$ where the pore pressure cannot be neglected. As the tensile strength of rock is usually small, fractures are apt to occur under the tensile stress. The maximum stress at the wall of cylindrical hole apparently takes place at the intersection between the wall and the axial plane which includes the axis of hole and the minimum transverse stress (maximum compression), that is, zy-plane. As $\gamma=90^\circ$ or 270° in the zy-plane:

$$\sigma_\rho = P$$

$$\sigma_z = \sigma_N + 2\nu (\sigma_{Hx} - \sigma_{Hy})$$

$$\sigma_\gamma = 3\sigma_{Hx} - \sigma_{Hy} - P \quad (43)$$

$$\tau_{z\gamma} = -2\tau_x$$

$$\tau_{\rho z} = \tau_{\rho\gamma} = 0$$

If the shearing stress $\tau_x=0, \sigma_\gamma, \sigma_z$ or σ_ρ may be the maximum at the wall of hole. If the stress σ_γ is maximum, new tension fractures may be formed along the axial plane, which includes the axis of revolution of cylindrical hole. If the stress σ_z is the maximum stress, new tension fractures may be formed along the transverse plane, which is perpendicular to the axis of revolution of cylindrical hole. In many cases, σ_N, σ_{Hx} and σ_{Hy} cannot be controlled, but we may control internal fluid pressure (P) up to the limit which is determined by the fracture strength, effective permeability and pumping rate. The preferable condition for the formation of axial fracture rather than transverse fracture is given by $\sigma_\gamma > \sigma_z$ as follows (Fig.3):

$$-P > -(3-2\nu)\sigma_{Hx} + (1-2\nu)\sigma_{Hy} + \sigma_N \quad (44)$$

where $\tau_x=0$. Even in the most difficult stress condition for the formation of axial fracture, $\sigma_{Hx} = \sigma_{Hy}$ and $\sigma_N = 0$ (except for tension), axial fractures are formed at:

$$-P > -2\sigma_H \quad (45)$$

If the shearing stress τ_x cannot be neglected in the plane which includes the axis of hole and the maximum transverse stress (minimum transverse compression), hydraulic fractures are formed along a plane which is inclined with the following angle θ from the axial plane (zy-plane):

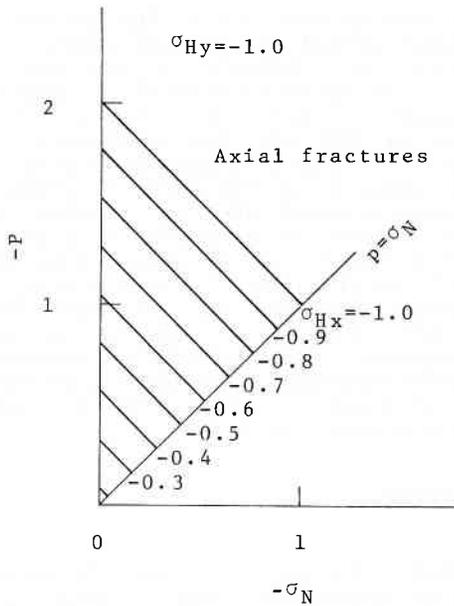


Fig.3 Conditions for the formation of axial fractures

$$\tan 2\theta = \frac{4\tau_x}{(3-2\nu)\sigma_{Hx} - (1-2\nu)\sigma_{Hy} - P - \sigma_N} \quad (46)$$

This equation suggests that the orientation of hydraulic fractures may be controlled with the internal fluid pressure. The control of fracture orientation can be theoretically used to estimate stresses in various orientations, although extremely quick loading is required to apply high fluid pressure in a borehole.

The normal stress σ_p to the wall of hole is equal to the internal fluid pressure (P). However the theoretical solutions indicate that the stress σ_p becomes tensional or less compressional at the just inner side of wall of rock to the direction of maximum transverse compression under the following stress condition:

$$\sigma_{Hx} - \sigma_{Hy} > -\frac{2}{3}(\sigma_{Hy} - P) \quad (47)$$

If the maximum stress in this inner extensional zone is higher than the pore pressure,

$$\frac{(3\sigma_{Hx} - \sigma_{Hy} - 2P)^2}{24(\sigma_{Hx} - \sigma_{Hy})} - P_p > 0 \quad (48)$$

then, arcuate fractures parallel to the wall of cylindrical hole may be formed. The arcuate spalling at the wall of cylindrical hole causes elongation of the circular section of borehole. The borehole elongation of vertical oil wells may be

used as indicator of the orientation of maximum horizontal compression in the earth's crust (Bell & Gough, 1981).

The stress state approaches to the primary stress state in short distances from the hole. For example, stresses are calculated at $\rho=3, \nu=0.3$ as follows.

$$\sigma_p = 0.89(\sigma_i + \sigma_m) + 0.11P$$

$$\sigma_z = \sigma_i - 0.13\sigma_d \cos 2\gamma$$

$$\sigma_\gamma = 1.11(\sigma_i + \sigma_m) - 0.11P - 1.04\sigma_d \cos 2\gamma \quad (49)$$

$$\tau_{\rho z} = 0.89(\tau_x \cos \gamma + \tau_y \sin \gamma)$$

$$\tau_{\rho z} = -1.19 \sin 2\gamma$$

$$\tau_{z\gamma} = 1.11(\tau_y \cos \gamma - \tau_x \sin \gamma)$$

The difference between the local stresses and regional stresses is only about 10-20% at the distance of one hole diameter from the wall of hole. Therefore, the fractures are expected to curve into the direction normal to the regional maximum stress in short distances from the hole.

The Geological Survey of Japan measured the in-situ stress by overcoring and hydraulic fracturing from an underground gallery in the Seikoshi gold mine, north western Izu Peninsula, Japan.

Results of in-situ stress measurement by overcoring are:

$$\sigma_1 = -2.1 \text{ MPa}, S78^\circ \text{ W}02^\circ \text{ dip}$$

$$\sigma_2 = -3.5 \text{ MPa}, S12^\circ \text{ E}17^\circ \text{ dip}$$

$$\sigma_3 = -17.4 \text{ MPa}, N04^\circ \text{ W}72^\circ \text{ dip}$$

Results of in-situ stress measurement by hydrofracturing are:

$$\sigma_1 = -3.4 \text{ MPa}, S44^\circ \text{ W}35^\circ \text{ dip}$$

$$\sigma_2 = -5.8 \text{ MPa}, S58^\circ \text{ E}17^\circ \text{ dip}$$

$$\sigma_3 = -12.3 \text{ MPa}, N10^\circ \text{ E}50^\circ \text{ dip}$$

The overcoring and hydraulic fracturing were conducted in the same boreholes. The host rock is diorite. The point of measurement is 350m deep below the surface at $34^\circ 54' \text{ N}$, $138^\circ 50' \text{ E}$. Although there are some differences between the results of stress measurements by overcoring and by hydrofracturing, both measurement suggest the essentially similar stress state of horizontal extension. The result of overcoring method shows the horizontal extension in the east-west direction probably due to the bending of the Philippine Sea plate which descends under the Eurasia plate along the Sagami Trough at just west of the Izu Peninsula.

Axial fractures were formed by hydraulic fracturing of a horizontal N57° E azimuth borehole in the Seikoshi mine, but the fractures soon turned into almost perpendicular direction to the regional minimum stress (Fig.4).

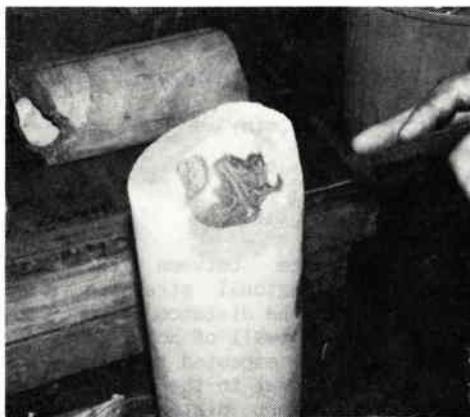


Fig.4 Overcored after hydraulic fracturing, Seikoshi mine.

The plastic tube stressmeter (Serata & Bellman, 1983) is suitable to control the stress field around a borehole as a flexible tube prevents the direct penetration of pressured fluid into the wall rock and as the deformation of hole and pressure can be measured precisely.

5. CONCLUSION

The exact stress distributions around a cylindrical hole or spheroidal cavity with internal fluid pressures were obtained from an analytical solution based on the three-dimensional theory of elasticity under various regional stress conditions. The critical internal fluid pressure for the formation of axial plane fractures is obtained from the theoretical solutions around a cylindrical hole. If the absolute value of internal fluid pressure is smaller than the absolute value of this critical pressure, transverse fractures may occur around a cylindrical hole. If the stress state in the transverse plane to the cylindrical hole is isotropic and all other stress components are zero, the critical internal fluid pressure for the formation of axial fracture is two times of the transverse stress. The hydraulic fractures are inclined by the shearing stress in the axial plane of hole which

includes the direction of the transverse maximum stress (minimum compression). The inclination of hydraulic fracture may be controlled by the internal fluid pressure. Although the exact control of fluid pressure is difficult due to fracturing of rock, the change of pumping speed of fluid may result in the change of effective internal pressure. The large deviation in the stresses in the perpendicular direction to the axis of cylindrical hole may cause arcuate parallel fractures at short distances from the wall of hole. This arcuate fractures cause the borehole elongation in the direction of transverse maximum compression. The internal pressure is effective to prevent the formation these arcuate fractures.

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