



# Analysis of stress, water flow and heat transfer in fractured rocks

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**Abstract:** To analyze the coupled field problem of stress, groundwater flow and heat transfer, the governing equations are formulated based in the integral form on the conservation law and them directly solved through discretization. It should be noted that direct discretization of the conservation law of various physical quantities using the "Rigid Bodies-Spring Model" makes the analysis not only simpler but also more flexible. Two coupled field problem of heat and groundwater flow as well as heat conduction and stresses are analyzed in this paper.

## 1 INTRODUCTION

Recently, the nuclear waste geological disposal, etc. have produced the problems of heat transfer and groundwater flow in fractured rocks. The analysis of heat and groundwater flow requires a evaluation of the effect of stress, and therefore it is necessary for this purpose to consider discontinuity of the rock mass in these analysis. Various approaches have been proposed to the stress analysis of fractured rocks, and the effect of discontinuity was simulated by using of the Cundall model (Cundall 1974), the joint element (Goodman et al. 1968), the rigid body spring model (Kawai. 1980), etc. Further Kyoya et al. 1985 and Oda et al. 1985 conducted an elaborate analysis in which discontinuity effect was considered in the constitutive equation by using damage tensor and fabric tensor. In the evaluation of the stability of the underground rock caverns, tanks of storing LPG, cracking always occur around the caverns due to contractive strain, and therefore a coupled analysis of heat transfer, groundwater flow and stress in cracked rocks is actually necessary.

In this paper direct discretization of the conservation law is proposed using the

RBSM to deal with these problems. As a matter of fact the RBSM can be considered a discrete model to fulfil the conservation law in the stress analysis. Under such consideration, extended use of the RBSM is attempted in these coupled field problems of fractured rock mass.

## 2 ANALYSIS OF THE COUPLED FIELD OF HEAT TRANSFER AND GROUNDWATER FLOW

### 2.1 Formulation

The governing partial differential equations of unsteady coupled field problem of heat transfer and groundwater flow can be given as follows (Utsugida 1985):

$$\rho c \frac{\partial T}{\partial t} + ((\rho c)_r v_i T)_{,i} = (\lambda T_{,i})_{,i} + Q \quad (1)$$

$$\left[ \frac{k}{\mu} (P_{,i} + (1 - \beta(T - T_0)) \rho g \delta_{i3}) \right]_{,i} = \frac{S_s}{\rho_f g} \frac{\partial P}{\partial t} \quad (2)$$

$$v_i = -\frac{k}{\mu} (P_{,i} + (1 - \beta(T - T_0)) \rho g \delta_{i3}) \quad (3)$$

Integrating them with respect to a region  $V$  and introducing Fourier's law;

$$q_i = -\lambda T_{,i} \quad (4)$$

and then rearranged using the Gauss' theorem, the equations (1) and (2) can be rearranged as follows;

$$\iiint_V \rho c \frac{\partial T}{\partial t} dv = -\iint_V (\rho c)_r v_i n_i T ds + \iint_V Q dv - \iint_V q_i n_i ds \quad (5)$$

$$\iint_{\mu} \frac{k}{\mu} (P_{,i} + \rho_f g \delta_{i3}) n_i ds \quad (6)$$

$$- \iint_{\mu} \frac{k}{\mu} \{ \rho_f g \beta (T - T_0) \delta_{i3} \} n_i ds = \iiint \frac{Ss}{\rho_f g \Delta t} \frac{\partial P}{\partial t} dv$$

where  $\rho c$ : Equivalent heat capacity of rock masses,  $(\rho c)_f$ : Heat capacity of fluid,  $T_0$ : Initial temperature,  $\beta$ : Thermal expansion coefficient of the fluid,  $v_i$ : Components of seepage velocity vector,  $\lambda$ : Thermal conductivity,  $Q$ : Heat generation rate,  $q_i$ : Components of heat flux,  $P$ : Pore water pressure,  $K$ : Permeability coefficient,  $\mu$ : Coefficient of fluid viscosity,  $g$ : Gravitational acceleration,  $\delta_{i3}$  (1 when  $i=3$ ),  $Ss$ : Specific storage,  $\rho_f$ : Fluid density,  $n_i$ : Directional cosine of the surface normal.

The equations (3), (5) and (6) are discretized and are to be solved simultaneously.

The following assumptions are made before discretization of the equation (5).

- 1) The temperature within elements is constant.
- 2) The heat flux on the interelement boundary surfaces is constant.
- 3) The seepage velocity on the interelement boundary surface is also constant.

Now let's assume that the volume of element  $i$  is  $V_i$ , the contact area with the adjacent element  $j$  is  $S_{ij}$ , and the element temperature is  $T_i$ . The temperature at the boundary surface is an average of temperatures at adjacent elements.

$$T_{ij} = (T_i + T_j) / 2 \quad (7)$$

Then the equation (5) is discretized as follows.

$$\rho_i c_i \dot{T}_i V_i = - \sum_j (\rho c)_f V_{n_{ij}} \left( \frac{T_i + T_j}{2} \right) S_{ij} + Q_i V_i - \sum_j q_{ij} S_{ij} \quad (8)$$

where  $V_{ij}$  and  $q_{ij}$  are the flow velocity and the heat flux at the boundary surface of elements in the direction of the outward normal, respectively. However the suffixes  $i$  and  $j$  in the equation (8) simply show the elements  $i$  and  $j$ , unlike the equations (1) through (6) which show a tensor expression.  $\sum$  implies summation of the adjacent elements. The heat flux  $q_{ij}$  is expressed by using the heat pass coefficient  $k_{ij}$  as follows.

$$q_{ij} = k_{ij} (T_j - T_i) \quad (9)$$

Here the definition of  $k_{ij}$  may be different balance according to the method of

heat to/from the adjacent element, as follows.

1) In the case of heat conduction:

$$k_{ij} = \lambda / \mu_{ij} \quad (10)$$

(  $\lambda$  = Thermal conductivity,  
 $\mu_{ij}$  = Distance between element  
gravitational centers )

2) In the case of convective heat

$$\text{transfer: } k_{ij} = \alpha \quad (11)$$

(  $\alpha$  : Coefficient of heat transfer )

3) In the case of thermal radiation:

$$k_{ij} = \sigma \epsilon F (T_i + T_j) (T_i^2 + T_j^2) \quad (12)$$

(  $\sigma$  : Stephan-Boltsman constant,

$\epsilon$  : Emissivity,

$F$  : Geometrical factor,

$T_i, T_j$  : Temperature of the elements  
 $i$  and  $j$  )

Introducing the mean load approximation to the time integration using a factor  $\theta$ , the equation (8) becomes as shown below.

$$\begin{aligned} & \left[ \rho_i c_i V_i + \theta \Delta t \sum_j \left( \frac{1}{2} (\rho c)_f V_{n_{ij}} + k_{ij} \right) S_{ij} \right] T_i^{t+\Delta t} \\ & + \left[ \theta \Delta t \sum_j \left\{ \left( \frac{1}{2} (\rho c)_f V_{n_{ij}} - k_{ij} \right) S_{ij} T_j^{t+\Delta t} \right\} \right] \\ & = \left[ \rho_i c_i V_i - (1-\theta) \Delta t \sum_j \left( \frac{1}{2} (\rho c)_f V_{n_{ij}} + k_{ij} \right) S_{ij} \right] T_i^t \\ & - \left[ (1-\theta) \Delta t \sum_j \left\{ \left( \frac{1}{2} (\rho c)_f V_{n_{ij}} - k_{ij} \right) S_{ij} T_j^t \right\} \right] + \Delta t Q_i^t V_i \end{aligned} \quad (13)$$

If  $\theta = 0$  in the equation, it will become the Euler method, which is unsuitable for the unsteady analysis to determine the temperature distribution over many hours as the equation will be divergent if a long time step is taken. Therefore the Crank-Nikorsen method ( $\theta = 0.5$ ) is adopted in the present analysis.

Similarly the equation (6) for unsteady groundwater flow is like the equation (5). The following assumptions are taken prior to the discretization.

- 1) The pore water pressure within elements is constant.
- 2) The pressure gradient on the interelement boundary surface is constant.

If the pressure gradient at the interelement boundary surface is equal to the quotient of the difference of the pore water pressure between the adjacent elements divided by the distance between their gravitational centers, the equation (6) become as follows.

$$\begin{aligned} & \left[ A_i V_i + \theta \Delta t \sum_j \frac{k_{ij}}{\mu_{ij}} S_{ij} \right] P_i^{t+\Delta t} \\ & + \left[ \theta \Delta t \sum_j \left( - \frac{k_{ij}}{\mu_{ij}} \right) S_{ij} P_j^{t+\Delta t} \right] \end{aligned} \quad (14)$$

$$\begin{aligned}
 &= [A_i V_i - (1-\theta) \Delta t \sum_j \frac{k_{ij}}{\mu l_{ij}} S_{ij}] P_i^t \\
 &- [(1-\theta) \Delta t \sum_j (-\frac{k_{ij}}{\mu l_{ij}}) S_{ij} P_j^t] + \Delta t \sum_j \frac{k_{ij}}{\mu} \rho_f g n_{z_{ij}} S_{ij} \\
 &- \Delta t \sum_j \frac{k_{ij}}{\mu} \rho_f g \beta (\frac{T_i^t + T_j^t}{2} - T_o) n_{z_{ij}} S_{ij}
 \end{aligned}$$

where  $n_{z_{ij}}$  is the vertical element of the outward unit normal vector. Next, the equation (3) for the flow velocity  $V_i$  obtained from the Darcy's law is discretized into the following equation.

$$\begin{aligned}
 v_{n_{ij}}^t = & -\frac{k_{ij}}{\mu} \left[ \frac{P_j^t - P_i^t}{l_{ij}} + \rho_f g n_{z_{ij}} \right. \\
 & \left. - \rho_f g \beta (\frac{T_i^t + T_j^t}{2} - T_o) n_{z_{ij}} \right]
 \end{aligned} \quad (15)$$

## 2.2 Numerical Results

Fig.1 shows an example of the analysis of groundwater flow in fractured rocks. This is the numerical example of a finite element analysis made by Kawamoto et al. 1977, indicating idealized lattice joint systems. The joint interval is 20m and the joint gap is 3mm. The permeability of each joint system can be obtained from  $k_f = g(2b)^2 / (12\nu) = 7 \times 10^{-2} \text{ cm/s}$  where (2b) is the gap between parallel model plates,

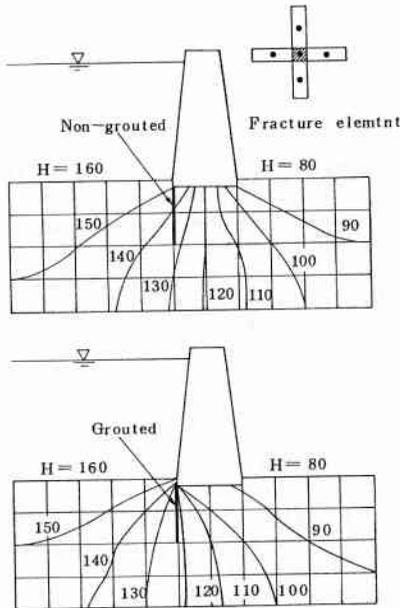


Fig.1 Analysis of groundwater flow in the dam foundation rock with joints

and is the kinetic viscosity coefficient. The permeability coefficient  $k_g$  of the grouted joint is  $k_g = 8 \times 10^{-5} \text{ cm/s}$ , if the opening of the grouted joint is assumed to be 0.1mm wide. The results of analysis are shown as the equipotential lines in the dam foundation rocks and are fairly good agreement with the Kawamoto et al. s results. Fig.2 shows an example of the coupled field analysis of heat transfer and groundwater flow in a simple two dimensional rock model. The analysis model is a 100 meter square and 10m thick model as shown in Fig.2a. with all the boundaries being impermeable, and the upper and lower surfaces made to be adiabatic boundaries, the model observes the occurrence and growth of the natural convection, and changes in the temperature distribution under initial rock temperature of 50°C with the left and the right boundary temperatures fixed at 50°C and 100°C respectively. Further, the static water pressure distribution is assumed at the initial condition. The material properties are the rock masses heat capacity:  $\rho c = 2.44 \times 10^6 \text{ J/m}^3\text{°C}$ , the fluid heat capacity:  $(\rho c)_f = 4.19 \times 10^6 \text{ J/m}^3\text{°C}$ , the rock thermal conductivity:  $\lambda = 2.9 \text{ W/m}^2\text{°C}$ , the rock specific storage:  $S_s = 3 \times 10^{-7} / \text{m}$ , the rock permeability:  $k = 10^{-5} \text{ m/s}$ , and the fluid thermal expansion coefficient:  $\beta = 0.00021 / \text{°C}$ .

In a fractured rock masses model, cracks were treated in such that no water flow within the joints was considered and the heat conductivity in the direction perpendicular to the joints was calculated as 1/100. It can be seen that the convection flow occurs and the temperature distribution changes 15 years later. When the vertical cracks initiate, the temperature distribution is considerably affected by them but no appreciable difference is observed in the calculation of the flow velocity vector.

In the present analysis, the basic equation for heat transfer and groundwater flow are expressed in the integral form, and discretized directly. This has enabled us to handle the problems in a similar way as in the analysis of the rigid body spring model, and find a clue to the coupled field analysis of heat transfer and the fluid flow under influence of crack initiation and growth.

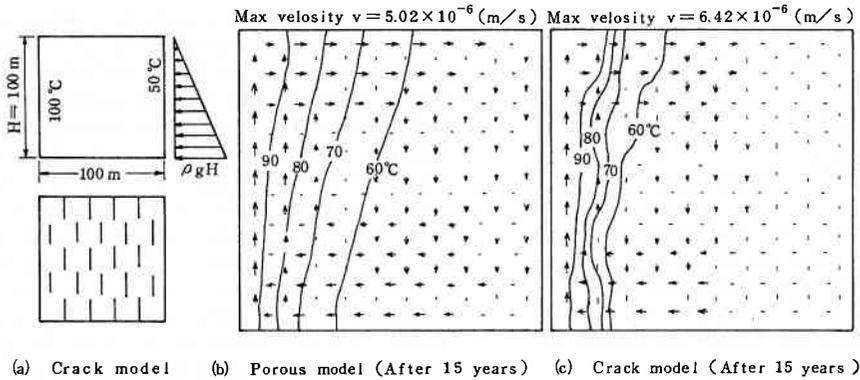


Fig.2 Analysis of heat convection flow in fractured rock masses

### 3 COUPLED FIELD ANALYSIS FOR HEAT CONDUCTION AND STRESSES

#### 3.1 Formulation of Elasto-Plastic Thermal Stress Analysis of Three-Dimensional Fractured Rocks

The elasto-plastic thermal stress analysis is here formulated by using the new discrete three-dimensional element proposed by Kawai 1980. In this new discrete element, the element mode is only considered at the centroid and arbitrary elements shape can be used. For example elements shown in Fig. 3 are considered. Each of them is assumed to be rigid, and their centroids of I and II are shown as points  $G_1$  and  $G_2$ . After deformation, the displacement  $\mathbf{u}$  of any arbitrary points within the rigid bodies I and II is shown by the following expressions.

$$\mathbf{u} = \begin{bmatrix} Q_1 & 0 \\ 0 & Q_2 \end{bmatrix} \begin{Bmatrix} U_1 \\ U_2 \end{Bmatrix} = [Q]\{U\} \quad (16)$$

$$U_i = (u_i, v_i, w_i, \theta_i, \phi_i, \chi_i) \quad (17)$$

$$Q_i = \begin{bmatrix} 1 & 0 & 0 & 0 & (z-z_i) & -(y-y_i) \\ 0 & 1 & 0 & -(z-z_i) & 0 & (x-x_i) \\ 0 & 0 & 1 & (y-y_i) & -(x-x_i) & 0 \end{bmatrix} \quad (18)$$

where  $U_i$  is the displacement vector at the centroid of the  $i$ th element, and  $Q_i$  is the  $(3 \times 6)$  matrix which depends on the coordinates components  $(x_i, y_i, z_i)$  at the centroid of the element and those  $(x, y, z)$  at any arbitrary point. The relative displacement of two adjacent elements occur at their interelement boundary surface after

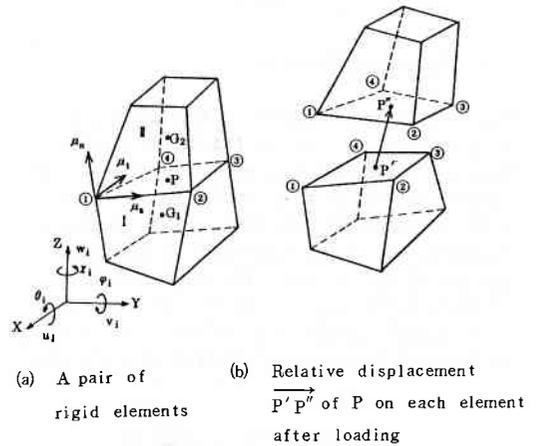


Fig.3 Elements of Rigid Body Spring Model

the deformation. It is assumed that in Fig. 3b, the point  $P$  on the boundary surface before the deformation move to  $P'$  and  $P''$  after the deformation. The relative displacement vector  $\delta = (\delta_x, \delta_y, \delta_z)$  can be shown by the following expression using the displacement  $U$  at the centroids of the adjacent elements I and II.

$$\{\delta\} = [Q_1, -Q_2] \begin{Bmatrix} U_1 \\ U_2 \end{Bmatrix} = [B]\{U\} \quad (19)$$

Further, the relative displacement vectors  $\{\delta\} = (\delta_n, \delta_s, \delta_t)$  in the directions of  $n, s$  and  $t$  in a system of rectangular coordinates having the normal vector to the boundary surface, as shown in Fig.3a, are given by the expression below using the coordinate transportation matrix  $T$ .

$$\{\delta\} = [T]\{\delta\} = [T][B]\{U\} = [A]\{U\} \quad (20)$$

Then, the strain vector  $\epsilon = (\epsilon_n, \epsilon_s, \epsilon_t)$  can be approximated by the expression below dividing the relative displacement vector  $\{\delta\}$  by the distance  $h$  between the centroids of the elements  $l$  and  $ll$ .

$$\{\epsilon\} = \{\delta\}/h = [A]\{U\}/h \quad (21)$$

The stress vector is:

$$\{\sigma\} = \begin{Bmatrix} \sigma_n \\ \tau_s \\ \tau_t \end{Bmatrix} = \begin{Bmatrix} k_n & 0 \\ k_s & \\ 0 & k_t \end{Bmatrix} \begin{Bmatrix} \epsilon_n \\ \epsilon_s \\ \epsilon_t \end{Bmatrix} = [D^e]\{\epsilon\} \quad (22)$$

where  $k_n$  and  $k_s$  and  $k_t$  are the normal stiffness and the shear stiffness, which are separately defined with regard to the continuous media and the discontinuous media (Hamajima et al. 1985).

Next, the formulation of the elasto-plastic thermal stress analysis is considered.

The incremental stages are assumed to be  $P^{(0)}, P^{(1)}, \dots, P^{(n)}$ . When the increment

$\Delta P^{(n)}$  is considered with the value of  $P^{(n)}$  stage assumed to be known, the virtual work is given by the following expression.

$$\iiint \delta \{\Delta \epsilon\}^T (\{\sigma\} + \{\Delta \sigma\}) dV - \iiint \delta \{\Delta u\}^T (\{\bar{F}\} + \{\Delta \bar{F}\}) dV - \iint \delta \{\Delta u\}^T (\{\bar{q}\} + \{\Delta \bar{q}\}) ds = 0 \quad (23)$$

where  $\bar{F}$  and  $\bar{q}$  represent the body force and the surface load.  $\{\epsilon\}$  and  $\{U\}$  are given by the expressions (21) and (16), and the incremental stress vector  $\{\Delta \sigma\}$  are given by the following expression.

$$\{\Delta \sigma\} = [D^{ep}] (\{\Delta \epsilon\} - \{\Delta \epsilon^T\}) \quad (24)$$

$[D^{ep}]$  is the elasto-plastic matrix which can easily be calculated if the yield function is given.  $\Delta \epsilon^T = (\beta(T - T_0), 0, 0)$  is the thermal strain vector,  $\beta$  is the thermal expansion coefficient to rock masses, and  $T_0$  is the initial temperature. Thus the equation (23) becomes as shown below.

$$\{\Delta f_s\} + \{\Delta f_v\} + \{\Delta f_r\} + \{\Delta r\} = [k^{ep}]\{\Delta u\}$$

$$\{\Delta f_r\} = \iint [A]^T [D^{ep}] \{\Delta \epsilon^T\} dV$$

$$\{\Delta r\} = \iint [Q]^T \{\bar{q}\} ds - \iint [Q]^T \{\bar{F}\} dv - \iint [A]^T \{\sigma\} ds \quad (25)$$

$$[k^{ep}] = \frac{1}{h} \iint [A]^T [D^{ep}] [A] ds$$

where  $\Delta f_s$  and  $\Delta f_v$  represent the surface loads and the body forces, and the balance  $\Delta r$  is assumed to be  $\Delta r = 0$ . Further, the surface integral to  $\{\Delta f_r\}$  and  $[k^{ep}]$  is

calculated by means of numerical integration.

### 3.2 Coupled Analysis of Heat Conduction and Stress

The following expression is used as the governing equation for heat conduction.

$$\iiint \rho c \frac{\partial T}{\partial t} dV = \iiint Q dV - \iint q_i n_i ds \quad (26)$$

Here, a coupled field analysis is made for unsteady heat conduction and stress. As the first step, the unsteady analysis of the governing equation for heat conduction is made, and based on the coupled temperature, an analysis is made on the thermal stress certain hours later. The Mohr-Coulomb's yield criterion for the constitutive equation of material was used.

$$f = \tau + \sigma \tan \phi - c = 0, \quad \sigma < \sigma_t \quad (27)$$

where  $\phi$  and  $C$  are the internal friction angle and the cohesion, and  $\sigma_t$  is the allowable tensile stress. equation (27) is constitutive equation with tension cut, which does not consider the influence of the temperature stress.

As an example, an analysis was made on the rock cavern storing LPG to which the finite element analysis by Ishizuka et al. 1985. Dry rock masses were assumed and their physical properties were:

the internal friction angle:  $\phi = 45^\circ$ ,  
Poisson's ratio:  $\nu = 0.3$ ,  
Young's modulus:  $E = 6.8 \times 10^4 \text{ kg/cm}^2$ ,  
the cohesion:  $C = 56 \text{ kg/cm}^2$ ,  
the allowable tensile stress:  $\sigma_t = 28 \text{ kg/cm}^2$   
density:  $\rho = 2.6 \text{ g/cm}^3$ ,  
specific heat:  $C_p = 0.753 \text{ KJ/kg}^\circ\text{C}$ ,  
the thermal conductivity:  $\lambda = 3.13 \text{ W/m}^\circ\text{C}$ ,  
and the coefficient of thermal expansion:  $\beta = 5 \times 10^{-6} / ^\circ\text{C}$

All of them were assumed to be constant irrespective of temperature.

The analysis results are as shown in Fig.4, which almost coincide with the Ishizuka et al. in respect to the thermal distribution characteristics. With regard to the tensile fracture region, however, the two results are not always the same as the tensile strength was assumed to be constant in this analysis although development of the tensile region are observed

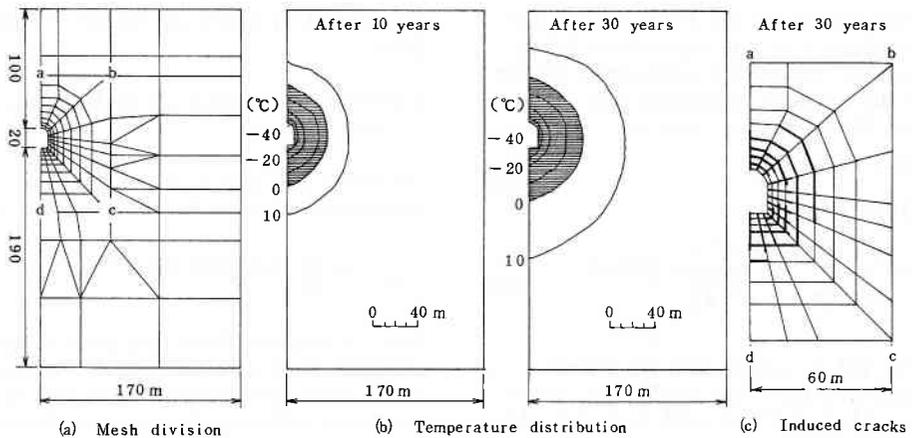


Fig.4 Temperature distribution and induced cracks in the rock cavern storing LPG

to be similar in both results. This analysis can be made changing the heat transfer mode within the cracks from conduction into convective or thermal radiation, at the same time tracing the development of cracks, and furthermore the three-dimensional analysis may be possible. It is therefore considered to be effectively applicable to LPG and LNG, in future.

#### 4 CONCLUSION

The RBSM can be applied successfully to analysis of crack as well as slip growth. The present authors examined possibility of application of this element to coupled field analysis of stresses and heat as well as water flow and they have drawn the following conclusions.

- 1) Concepts of the RBSM element can be equally applied to the heat transfer as well as the seepage flow analysis. Naturally element of any shape can be used and coupling effect of heat transfer and crack growth may be successfully studied.
- 2) Consideration of flow in the crack as well as along to vertical direction of the crack can be made in the heat transfer and seepage flow analysis.
- 3) Coupled field analysis of heat transfer and seepage flow may be possible under the influence of crack initiation and growth in discontinuous rock masses.
- 4) Coupled analysis of stresses and heat transfer may be possible for the 3D discontinuous rock foundation, similarly coupled analysis of stresses and seepage may be expected in near future.

#### REFERENCES

- Cundall, P.A. 1971. A computer model for simulating progressive large-scale movements in blocky rock system, symp. on rock fracture, Nancy, Sec.2-B.
- Goodmann, R.E., Tayler, R.I. and Brekke, T.L. 1968. A model for the mechanics of jointed rock, J. of soil mech. and found. Proc. A.S.C.E. SM3, P.637-659.
- Hamajima, R., Kawai, T., Yamashita, K. and Kusabuka, M. 1985. Numerical analysis of cracked and jointed rock mass. Fifth int. conf. on num. meth. in geomech. P.207-214.
- Ishizuka, Y., Kinoshita, N., and Okuno, T. 1985. Stability analysis of a rock cavern with LPG storage under thermal stresses. Fifth int. conf. on num. metho. in geomech. P.1233-1240.
- Kawai T. 1980. some considerations on the Finite Element Method. Int. J. for num. meth in engr. vol.16, P.81-120.
- Kawamoto, T. and Kadota, S. 1977. Analysis of seepage flow in jointed rock masses. Hydro Electric Power, No.147. P.7-16.
- Kyoya, T., Ichikawa, Y. and Kawamoto, T. 1985. A damage mechanics theory for discontinuous rock mass. Fifth int. conf. on num. meth. in geomech. P.469-480.
- Oda, M. and Maeshibu, T. 1985. Characterization of jointed rock masses by fabric tensor. Fifth int. con. on num. meth. in geomech. P.481-488.
- Utsugida, Y. 1985. Coupled analysis of flow and heat around a high-level nuclear waste repository. Fifth int. conf. on num. meth. in geomech. P.711-716.

PROCEEDINGS OF THE SIXTH INTERNATIONAL CONFERENCE ON NUMERICAL  
METHODS IN GEOMECHANICS / INNSBRUCK / 11-15 APRIL 1988

# Numerical Methods in Geomechanics Innsbruck 1988

*Edited by*

G.SWOBODA

*Institute of Structural Engineering, University of Innsbruck*

OFFPRINT



*Published on behalf of the International Committee for Numerical Methods in Geomechanics by*

A.A.BALKEMA / ROTTERDAM / BROOKFIELD / 1988